Ultra Low Phase Noise DDS

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ABSTRACT

Direct Digital Synthesizers traditionally use a phase accumulator and a phase-to-amplitude conversion mechanism to form complex samples of arbitrary frequency sinusoids. The CORDIC [1] algorithm is the most common phase-to-amplitude conversion processes. To obtain low levels of phase noise with a small number of iterations the DDS often employs a two pass algorithm in which the complex samples formed from high order phase bits are corrected by post processing with terms derived from low order phase bits. This paper presents a modified version of the CORDIC based DDS that suppresses the amplitude noise generated by the second pass phase correction. We then show that the amplitude noise suppression is equivalent to an embedded AGC. We then recast the second order normal recursive filter as a recursive version of the CORDIC and insert the equivalent AGC to stabilize the loop against finite arithmetic and signal growth due to the CORDIC. We show this to be a very interesting variation of the DDS.

1. INTRODUCTION

Many DSP algorithms in communication systems require values of a complex sinusoid (sine and cosine) of specific angles to accomplish a particular processing task. Examples include the DFT (discrete Fourier transform), the FFT (fast Fourier transform), digital up converters, digital down converters, and carrier recover loops [2]. Algorithms implemented in a high level language by a main frame or personal computer may compute the required sines and cosines from series expansions via a subroutine call as they are needed. Algorithms embedded in an ASIC (Application Specific Integrated Circuit) or in a fixed point microprocessor or FPGA can not afford the luxury of a subroutine call to a series expansion. They require another method to obtain values of the complex exponential for the specified value of argument.

In some algorithms, such as in the FFT, the values of the sines and cosines are known before hand and may be pre-computed and stored in a trig table. In other algorithms, the range of arguments may be too numerous to use a pre-computed table. We can still use the pre-computed table if we are willing to allow an acceptable approximation to the specified angle of the sine and cosine.

Let us examine the system effects of using approximations to the desired angles. The DDS (Direct Digital Synthesizer) is the standard mechanism to form a complex time series representing sample values of a sine and cosine. Figure 1 is the block diagram of a simple DDS formed by a 48-bit phase accumulator, a quantizer that extracts the 10 most significant bits from the accumulator, and a look up table addressed by the quantized 10-bit field to perform the phase to complex sinusoid conversion.

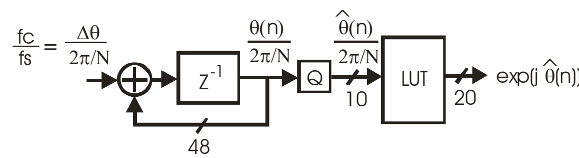

Figure 1. DDS: Phase Accumulator, Quantizer, and LUT

It is easy to verify [3] that the sequence of phase angle errors formed by the b-bit quantized address is a periodic sawtooth with amplitude proportional to the LSB, 2^{-b}. It is similarly easy to verify that the spectrum of the sinusoid formed with a sawtooth phase error contains a set of spurious spectral lines with the largest line 2^{-b} below the carrier spectral line. The largest spurious line is 6b dB below the carrier, where b is the quantized address width accessing the look up table. The signal formed with the 10-bit address shown in figure 1 should exhibit a spur 60 dB below the carrier. Figure 2, subplot(2,1,1) shows the -60 dB spur levels of a sinusoid formed with a 10-bit address while subplot(2,1,2) shows the reduction in spur level due to random dithering of the accumulator output prior to quantization.

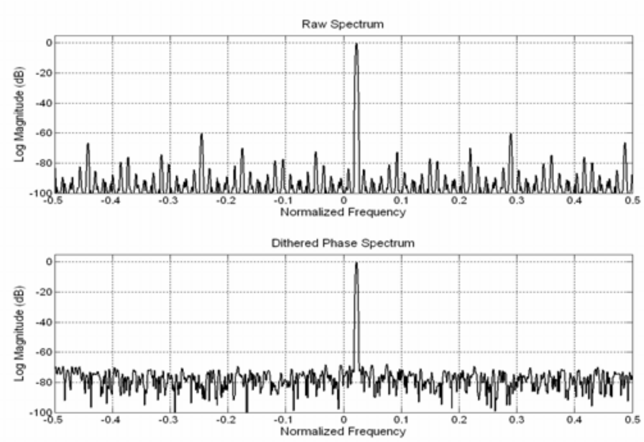

Figure 2. Spectra of 10-bit Angle DDS Output without and with Random Dither

To obtain spur levels 100 dB below the carrier we would require a 17-bit address for a table containing 131072 entries. The symmetry of sines and cosines allow some memory savings by storing the sinusoid samples in the first quadrant. Nevertheless, we are thus faced with the problem of doubling the table size for every 6-dB decrease in spurious level.

We can avoid the need for large tables by using the CORDIC algorithm to implement a set of elementary rotations requiring only shift and add operations and a look up table containing only b entries. Here we exchange a processing task for memory. The spur level of the time series formed by the CORDIC still has spurs 6b dB below the carrier but now b represents the number of rotation cycles implemented in the CORDIC. Figure 3 shows a modified DDS structure in which the length 2^b LUT is replaced by the CORDIC algorithm supported by a length b LUT. We delay detailed discussion of the CORDIC till a later section of this paper.

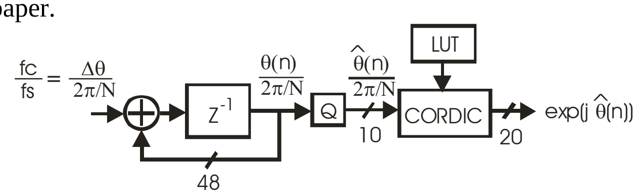

Figure 3. DDS: Phase Accumulator, Quantizer, and CORDIC

2. SPURIOUS LEVEL SUPPRESSION

The quantization of the 48 bit accumulator to obtain the 8-bit address introduces a phase error term in the angle presented to the angle to amplitude conversion whether the conversion is performed by the look-up table or the CORDIC algorithm. The effect of the angle approximation is seen in (1): the angle delivered to the trig function is the angle we want plus an error angle proportional to the bits left behind in the accumulator. Substituting the approximate angle in the trig function and recognizing that the error angle is small, on the order of $2\pi\ 2^{-10}$, we can use the small angle approximation to extract the phase error term from the trig function as shown in (2).

$$\hat{\theta}(n) = \theta(n) + \theta_{Error}(n) \qquad (1)$$

$$\begin{aligned} e^{j\hat{\theta}(n)} &= e^{j[\theta(n)+\theta_{Error}(n)]} \\ &= e^{j\theta(n)}\, e^{j\theta_{Error}(n)} \\ &\cong e^{j\theta(n)}[1 + j\,\theta_{Error}(n)] \\ &= e^{j\theta(n)} + j\,\theta_{Error}(n)\, e^{j\theta(n)} \end{aligned} \qquad (2)$$

The results seen here is that, even with the phase angle errors, the desired carrier is present in the output which contains a second term seen to be the phase angle time series phase modulated to the carrier center frequency. It is this second term that is responsible for the spurs in the signal spectrum. Of course we can reduce the spur levels by reducing the amplitude of the phase error terms. This can be accomplished by using a wider bit field to represent the approximate phase angles.

An alternate approach is to recognize that the phase angle error is not random but, for this example, is $-2\pi/2^{48}$ times the bit field left behind by the quantization process. We can extract the error term by scaling the difference between the input and the output of the quantizer and use this term to obtain an improved estimate of the desired trig function values from the trig values formed with the quantized value of the angle. This relationship is seen in (3).

$$\theta_{Error}(n) = \theta(n) - \hat{\theta}(n) \qquad (3)$$

The feed forward correction applied to the output of the trig function is seen in (4). In a sense, this is the cosine and sine of the sum of two angles where we have replaced the cosine and sines of the small angle θ_{Error} by 1 and by and θ_{Error} respectively. A block diagram of the feed forward processing is shown in figure 4.

$$\begin{aligned} e^{j\hat{\theta}(n)} &= e^{j\theta(n)}\, e^{j\theta_{Error}(n)} \\ e^{j\theta(n)} &= e^{j\hat{\theta}(n)} e^{-j\theta_{Error}(n)} \\ &\cong e^{j\theta(n)} - j\,\theta_{Error}(n)\, e^{j\theta(n)} \\ &= [\cos(\hat{\theta}(n)) + \theta_{Error}(n)\sin(\hat{\theta}(n))] \\ &\quad + j\,[\sin(\hat{\theta}(n)) - \theta_{Error}(n)\cos(\hat{\theta}(n))] \end{aligned} \qquad (4)$$

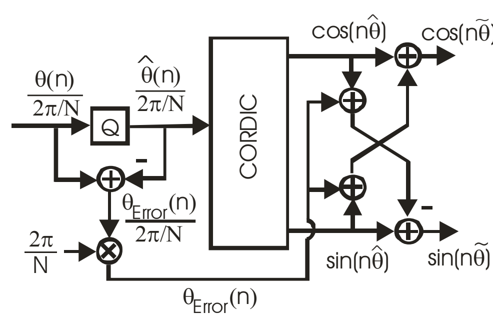

Figure 4. Block Diagram Feed Forward Angle Correction

The effect of the feed forward angle correction on the spectrum of the DDS output is quite dramatic. Figure 5, subplot (2,1,1) shows the spectrum formed with a 10 bit angle resolution corrected by the feed forward process of (4). Note the change in the y-axis scaling of figure 5 and figure 2, of -150 dB and -100 dB respectively. Note that the -60 dB spur level of figure 2 has been suppressed by the angle feed forward to -110 dB. We would have expected -